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ON FREE CONVECTION IN A HEAT-RELEASING GRANULAR BED

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Free convection in a heat-releasing granular bed for constant and exponential forms of heat release has been mathematically modeled on the basis of a two-temperature model. The extremum character of the dependence of the flow rate of the gaseous heat-transfer agent on the heat-release power and bed "choking" (cessation of the heat-transfer-agent filtration) for higher-than-average heat releases have been established. Dimensionless dependences for calculation of the flow rate of the heat-transfer agent and its outlet temperature in the region of stable convection have been obtained.

Keywords: *granular bed, free (natural) convection, bed choking, two-temperature model, heat-transfer agent.*

Introduction. Free (natural) convection is known to be one of the most widely occurring hydrodynamic phenomena in nature [1]. In particular, it is observed in granular and porous media under certain conditions; therefore, one faces the problem of natural convection in corresponding fields of science and technology (geophysics, soil mechanics, rheology, foundry practice, production of insulating materials, etc. [2]). A special place is occupied by free convection in heat-releasing granular beds (beds of different biological origin, a semitransparent-particle bed on which solar radiation is incident). We emphasize that the latter of the mentioned disperse systems offers the basic component of highly efficient solar-energy heaters. In connection with the poor understanding of the phenomenon of free convection in a heat-releasing granular bed, we have sought to investigate its regularities for constant and exponential forms of heat release.

Formulation of the Problem. We consider a heat-releasing granular bed that is in a stationary heat-transfer agent of fairly large volume with a constant temperature T_0. Heating of the heat-transfer agent in the bed leads to its free convection. To describe this phenomenon we use the well-known two-temperature model of a granular bed. The corresponding system of equations and boundary conditions in the one-dimensional case appears as

$$c_f \varepsilon \rho_f v \frac{dT_f}{dx} = \frac{d}{dx}\left(\varepsilon \lambda_f \frac{dT_f}{dx}\right) + \frac{6(1-\varepsilon)\alpha}{d}(T_s - T_f) , \tag{1}$$

$$0 = \frac{d}{dx}\left((1-\varepsilon)\lambda_s \frac{dT_s}{dx}\right) + \frac{6(1-\varepsilon)\alpha}{d}(T_f - T_s) + Q(1-\varepsilon) , \tag{2}$$

$$\rho_f v \frac{dv}{dx} = -150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu_f u}{d^2} - \rho_{f0} g + \rho_{f0} g \beta (T_f - T_0) - \frac{dp}{dx} + \mu_f \frac{d^2 v}{dx^2} , \tag{3}$$

$$\frac{d}{dx}(\rho_f v) = 0 ; \tag{4}$$

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$$x = 0: \quad p = \rho_{f0} g H + p_{atm}, \quad \frac{dv}{dx} = 0, \tag{5}$$

$$\varepsilon c_f \rho_f v\,(T_f - T_0) = (1 - \varepsilon)\,\lambda_s \frac{dT_s}{dx} + \varepsilon \lambda_f \frac{dT_f}{dx}, \tag{6}$$

$$(1 - \varepsilon)\,\lambda_s \frac{dT_s}{dx} = \alpha_0\,(T_s - T_0); \tag{7}$$

$$x = H: \quad p = p_{atm}, \quad \frac{dT_f}{dx} = 0, \quad \frac{dv}{dx} = 0, \tag{8}$$

$$(1 - \varepsilon)\,\lambda_s \frac{dT_s}{dx} = \alpha_H\,(T_f - T_s). \tag{9}$$

The density of the heat-transfer agent as a function of temperature is represented in standard form: $\rho_f = \rho_{f0}(1 - \beta(T_f - T_0))$. The influence of the pressure on the heat-transfer-agent density is disregarded. The force of friction of the heat-transfer agent against particles in (3) has been represented as the linear form of the well-known Ergun equation [3].

Reduction to a Dimensionless Form. The procedure of making the system of equations (1)–(9) dimensionless is performed in a standard manner. The quantity $v^* = \sqrt{gH}$ is taken as the characteristic velocity (velocity scale) of the heat-transfer agent. Following [4], we represent the term $-\rho_{f0}g$ in (3) as the gradient of hydrostatic pressure $\frac{dp_0}{dx}$ in the stationary heat-transfer agent of density ρ_{f0}. Taking account of this fact and of the smallness of the heat-transfer-agent velocity, we write system (1)–(9) in dimensionless form

$$(1 - \beta T_0 \theta_f)\,\varepsilon v' \frac{d\theta_f}{d\xi} = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_f}\frac{d\theta_f}{d\xi}\right) + \frac{1}{\mathrm{Pe}}\,(\theta_s - \theta_f), \tag{10}$$

$$0 = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_s}\frac{d\theta_s}{d\xi}\right) + \frac{1}{\mathrm{Pe}}\,(\theta_f - \theta_s) + \overline{Q}, \tag{11}$$

$$(1 - \beta T_0 \theta_f)\,\mathrm{Re}^2\, v' \frac{dv'}{d\xi}$$

$$= \mathrm{Gr}\,\theta_f - \frac{H^2 p_{atm}}{\rho_{f0} v_{f0}^2}\frac{d\,(p' - p_0/p_{atm})}{d\xi} - 150\left(\frac{1 - \varepsilon}{\varepsilon}\right)^2\left(\frac{H}{d}\right)^2\frac{\mu_f}{\mu_{f0}}\,\mathrm{Re}\,v' + \frac{\mu_f}{\mu_{f0}}\,\mathrm{Re}\frac{d^2 v'}{d\xi^2}, \tag{12}$$

$$\frac{d}{d\xi}\left((1 - \beta T_0 \theta_f)\,v'\right) = 0, \tag{13}$$

$$\xi = 0: \quad p' = \frac{\rho_{f0} g H}{p_{atm}} + 1, \quad (1 - \beta T_0 \theta_f)\,\varepsilon v' \theta_f = \frac{1}{\mathrm{Pe}_f}\frac{d\theta_f}{d\xi} + \frac{1}{\mathrm{Pe}_s}\frac{d\theta_s}{d\xi}, \quad \frac{d\theta_s}{d\xi} = \frac{\alpha_0 H}{(1 - \varepsilon)\,\lambda_s}\,\theta_s, \quad \frac{dv'}{d\xi} = 0, \tag{14}$$

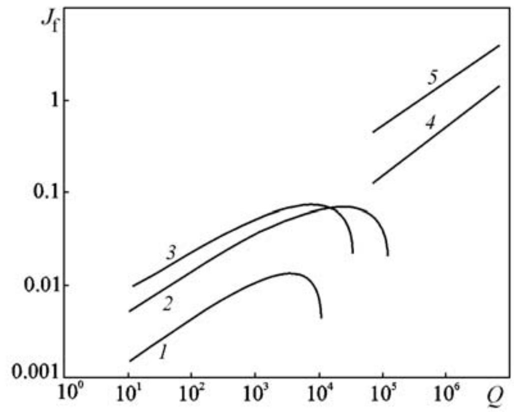

Fig. 1. Specific flow rate of the heat-transfer agent vs. heat-release power for Q = const (1–3, air; 4 and 5, water): 1) d = 10, 2 and 3) 40, 4) 1, 5) 4 mm; 1 and 2) H = 2, 3) 8, 4 and 5) 0.2 m. T_0 = 290 K. J_f, kg/(m^2·sec); Q, W/m^3.

$$\xi = 1: \quad p' = 1\,, \quad \frac{d\theta_f}{d\xi} = 0\,, \quad \frac{dv'}{d\xi} = 0\,, \quad \frac{d\theta_s}{d\xi} = \frac{\alpha_H H}{(1-\varepsilon)\,\lambda_s}(\theta_f - \theta_s)\,. \tag{15}$$

Analysis of the Obtained Results. In solving system (10)–(15) numerically, we have taken the values of the parameters λ_f, λ_s, α, α_0, and α_H in accordance with the recommendations of [5–7].

Heat Source of Constant Strength (Q = const). The mass fluxes of the heat-transfer agent for air and water are shown in Fig. 1. We note that the calculations for water have been carried out for such Q values at which $t_f \leq 100^{\circ}$C. As is seen, the mass flux of the heat-transfer agent markedly grows with particle diameter, which is attributed to the decrease in the interphase-interaction force. The heat-release power exerts a substantial influence on J_f. Calculations for air, which encompass a large range of variation in Q, show the extremum character of the $J_f(Q)$ plot. The sharp decrease in the mass flux of the heat-transfer agent when $J_f > (J_f)_{max}$ allows the conclusion on bed "choking" at higher-than-average Q, i.e., on the cessation of convection of the heat-transfer agent and on passage to an unbounded heating of the bed. Such a character of the $J_f(Q)$ plot can be explained, from Eq. (3), by the influence of two completing factors: the gas–particle resistance force $\left(-150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu_f u}{d^2}\right)$ that grows with T_f (and consequently Q) and the buoyancy force $\rho_{f0} g \beta (T_f - T_0) = \rho_{f0} g \frac{T_f - T_0}{T_f}$ whose growth with T_f is bounded by $\rho_{f0} g$. To evaluate $(J_f)_{max}$ we can use the condition of equality of these forces

$$150\,\frac{(1-\varepsilon)^2}{\varepsilon^3}\,\frac{\mu_f(T_f)\,u}{d^2} = \rho_{f0} g\,\frac{T_f - T_0}{T_f}\,. \tag{16}$$

Equation (16) for $\frac{T_f - T_0}{T_f} \to 1$ yields

$$(\mathrm{Re}_d)_{max} = \frac{(J_f)_{max} d}{\mu_{f0}} \approx \frac{\varepsilon^3}{150\,(1-\varepsilon)^2}\,\frac{g d^3}{\nu_{f0}\nu_f(T_f)}\,. \tag{17}$$

For $\varepsilon = 0.4$, we have $\frac{\varepsilon^3}{150(1-\varepsilon)^2} \approx 0.0012$. Processing of $(J_f)_{max}$ values calculated from (17) leads to a similar dependence:

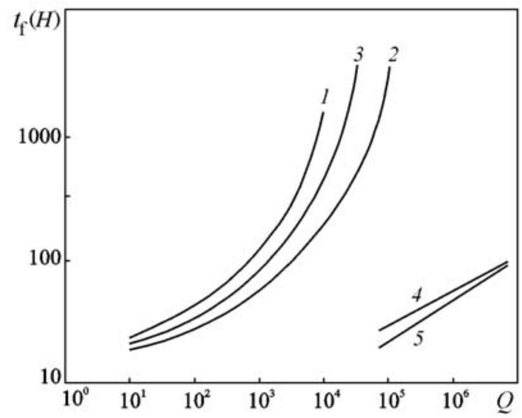

Fig. 2. Temperature of the heat-transfer agent at exit from the bed vs. heat-release power Q = const. Notation 1–5 is the same as that in Fig. 1. t_f, ^oC; Q, W/m^3.

$$(\mathrm{Re_d})_{\max} = 0.001\left(\frac{gd^3}{\nu_{f0}^2}\right)^{0.8}. \tag{18}$$

For Q values corresponding to $(J_f)_{\max}$, we obtain the formula

$$\overline{Q} = 2\cdot 10^{-5}\left(\frac{gd^3}{\nu_{f0}^2}\right)^{0.34}, \tag{19}$$

which points to the unique relationship between $(\mathrm{Re_d})_{\max}$ and $\overline{Q}$. For stable-convection regimes (left-hand branch of the function $J_f(Q)$), we establish the generalized dependence

$$\mathrm{Re_d} = 0.7\,\mathrm{Gr}_*^{0.55}\left(\frac{H}{d}\right)^{-1.6}, \tag{20}$$

which approximates J_f values with a standard deviation of 27.7%. The checking domain of (20) is $3\cdot 10^4 \le \mathrm{Gr}_* \le 5.7\cdot 10^{10}$, $50 \le \frac{H}{d} \le 200$. We emphasize that the obtained values of the exponents of Gr_* and $\frac{H}{d}$ in (20) make it possible to establish an extremely simple dependence equivalent to (20):

$$\mathrm{Re}_H = 0.67\,\mathrm{Gr_d}^{0.55}. \tag{21}$$

Figure 2 shows the outlet heat-transfer-agent temperatures $t_f(H)$ whose knowledge is important in the practical implementation of the systems in question. Generalization of $T_f(H)$ for the above conditions can easily be obtained from (20) or (21) and from the equation of total heat balance of the system

$$c_f J_f\,(T_f\,(H) - T_0) = Q\,(1-\varepsilon)\,H. \tag{22}$$

For $\theta_f(1)$, we have

$$\theta_f\,(1) = \frac{Q\,(1-\varepsilon)\,H}{c_f J_f T_0} = 1.5\,\frac{Q\,(1-\varepsilon)\,H^2}{c_f \mu_{f0} T_0}\,\mathrm{Gr_d}^{-0.55}. \tag{23}$$

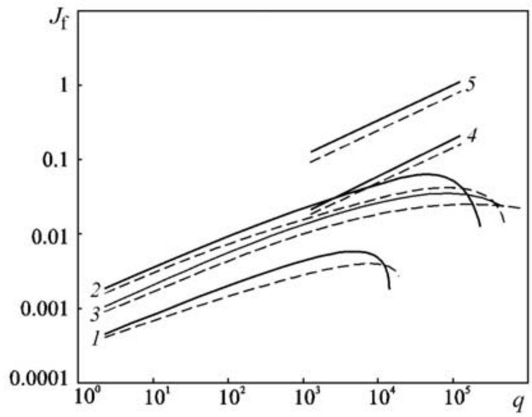

Fig. 3. Specific flow rate of the heat-transfer agent vs. incident radiation flux for $Q = \frac{qB}{d(1-\varepsilon)} \exp\left(-B\frac{(H-x)}{d}\right)$. Solid curves, $B = 0.1$; dashed curves, $B = 0.33$. Notation 1–5 is the same as that in Fig. 1. q, W/m^2; J_f, kg/(m^2·sec).

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Heat Source of Variable Strength $\left(Q = \frac{qB}{d(1-\varepsilon)} \exp\left(-B\frac{(H-x)}{d}\right)\right)$. Such a source type is implemented in the case where the radiant-energy flux of value q is incident on the upper boundary of the bed of semitransparent particles [7]. The calculated values of the mass flux of the heat-transfer agent are shown in Fig. 3. Their character is generally analogous to the case of a constant heat-release power (Fig. 1). We note only one fundamental difference: J_f increases with bed height for Q = const (curves 2 and 3 in Fig. 1), whereas in the case of an exponential heat source J_f values decrease with growth in H (curves 2 and 3 in Fig. 3). The character of change in J_f on attainment of the maximum values also points to bed "choking" with further increase in q. Generalization of the $(J_f)_{max}$ values is obtained in the form

$$(\mathrm{Re_d})_{max} = 0.0022 \left(\frac{gd^3}{\nu_{f0}^2}\right)^{0.8} \left(\frac{H}{d}\right)^{-0.4} B^{-0.32}. \tag{24}$$

For calculation of the corresponding values of the incident radiation fluxes, we establish the dependence

$$\bar{q} = 2.4 \cdot 10^{-5} \left(\frac{gd^3}{\nu_{f0}^2}\right)^{0.55} B^{0.5}. \tag{25}$$

Generalization of the calculated values of J_f under stable-convection conditions (left-hand branch of the $J_f(q)$ function) is obtained in a form analogous to (20):

$$\mathrm{Re_d} = 1.5\, (\mathrm{Gr}'_*)^{0.55} \left(\frac{H}{d}\right)^{-1.8} B^{-0.2}. \tag{26}$$

Formula (26) approximates J_f values with a standard error of 26% and has been checked in the following interval: $8.7 \cdot 10^3 \le \mathrm{Gr}'_* \le 10^9,\ 50 \le \frac{H}{d} \le 200$.

Figure 4 shows $t_f(H)$ values. Just as in the case of a constant-strength source, their generalization can be obtained with the heat-balance equation

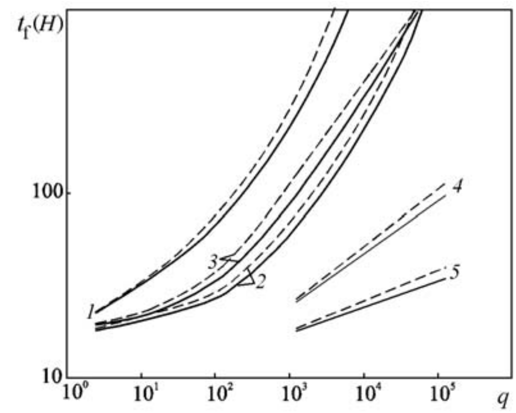

Fig. 4. Temperature of the heat-transfer agent at exit from the bed vs. incident radiation flux for $Q = \dfrac{qB}{d(1-\varepsilon)} \exp\left(-B\dfrac{(H-x)}{d}\right)$. Solid curves, $B = 0.1$; dashed curves, $B = 0.33$. Notation 1–5 is the same as that in Fig. 1. t_f, ^{o}C; q, W/m^2.

$$c_f J_f (T_f(H) - T_0) = q \tag{27}$$

and dependence (26)

$$\theta_f(1) = 0.67 \frac{qd}{c_f \mu_{f0} T_0} (\mathrm{Gr}'_*)^{-0.55} \left(\frac{H}{d}\right)^{1.8} B^{0.2} . \tag{28}$$

Conclusions. In the unified methodological context, we have considered the phenomenon of free convection in the heat-releasing granular bed for different forms of heat release. The extremum character of the dependence of the flow rate of the gaseous heat-transfer agent on the heat-release power and bed "choking" (cessation of the filtration of the heat-transfer agent) for higher-than-average heat release have been established in all cases. Within the framework of similarity theory, we have obtained the generalized dependences for calculation of the mass flow rate of the heat-transfer agent (20), (21), and (26) and of $(J_f)_{max}$ values and the corresponding heat-release powers (18), (19), (24), and (25).

NOTATION

B, radiation-attenuation coefficient; c_f, specific heat of the heat-transfer agent at constant pressure, J/(kg·K); d, particle diameter, m; g, free-fall acceleration, m/sec^2; $\mathrm{Gr} = \dfrac{g\beta H^3 T_0}{\nu_{f0}^2}$, $\mathrm{Gr_d} = \dfrac{g\beta_0 H^3 Q(1-\varepsilon)H}{\nu_{f0}^2 c_f \rho_{f0}\sqrt{gH}} \left(\dfrac{d}{H}\right)$, $\mathrm{Gr}_* = \dfrac{g\beta_0 H^3 Q(1-\varepsilon)H}{\nu_{f0}^2 c_f \rho_{f0}\sqrt{gH}}$, and $\mathrm{Gr}'_* = \dfrac{g\beta_0 H^3 q}{\nu_{f0}^2 c_f \rho_{f0}\sqrt{gH}}$, Grashof numbers; H, height of the granular bed, m; $J_f = \rho_f \varepsilon v = \rho_{f0} u_0$, specific mass flow of the heat-transfer agent, kg/(m^2·sec); p, pressure, Pa; p_0, hydrostatic pressure ($p_0 = g\rho_{f0}(H-x) + p_{atm}$), Pa; $p' = p/p_{atm}$; $\mathrm{Pe} = \dfrac{c_f \rho_{f0} v^* d}{6(1-\varepsilon)\alpha H}$, $\mathrm{Pe_f} = \dfrac{c_f \rho_{f0} v^* H}{\varepsilon \lambda_f}$, and $\mathrm{Pe_s} = \dfrac{c_f \rho_{f0} v^* H}{(1-\varepsilon)\lambda_s}$, Péclet numbers; q, incident radiation flux, W/m^2; $\overline{q} = \dfrac{q}{c_f \rho_{f0} v^* T_0}$; Q, heat-source strength, W/m^3; $\overline{Q} = \dfrac{Q(1-\varepsilon)H}{c_f \rho_{f0} v^* T_0}$; $\mathrm{Re} = \dfrac{\rho_{f0} v^* H}{\mu_{f0}}$, $\mathrm{Re_d} = \dfrac{J_f d}{\mu_{f0}}$, and $\mathrm{Re}_H =$

$\frac{J_f H}{\mu_{f0}}$, Reynolds numbers; T_f and T_s, temperature of the heat-transfer agent and particles, K; T_0, temperature of the stationary heat-transfer agent outside the bed, K; u, rate of filtration of the heat-transfer agent, m/sec; v, velocity of the heat-transfer agent in the gaps between particles ($v = u/\varepsilon$), m/sec; $v' = v/v^*$; $v^* = \sqrt{gH}$; x, longitudinal coordinate, m; α, α_0, and α_H, heat-exchange coefficients, W/(m^2·K); β, temperature coefficient of volumetric expansion (for air, $\beta = 1/T_f$ and $\beta_0 = 1/T_0$), 1/K; ε, porosity; $\theta_f = (T_f - T_0)/T_0$; $\theta_s = (T_s - T_0)/T_0$; λ_f, λ_s, longitudinal thermal conductivities of the heat-transfer agent and the skeleton from particles, W/(m·K); μ_f, dynamic viscosity of the heat-transfer agent, kg/(m·sec); ν_f, kinematic viscosity of the heat-transfer agent, m^2/sec; $\xi = x/H$; ρ_f, density of the heat-transfer agent, kg/m^3. Subscripts: atm, atmospheric; f, heat-transfer agent; H, at the granular-bed height; max, maximum; s, particles; 0, at entry into the bed.

REFERENCES

1. O. G. Martynenko and Yu. A. Sokovishin, *Free-Convective Heat Transfer: Handbook* [in Russian], Nauka i Tekhnika, Minsk (1982).
2. B. Gebhart, Y. Jaluria, R. L. Mahajan, and B. Sammakia, *Buoyancy Induced Flows and Transport* [Russian translation], Vol. 1, Moscow (1991).
3. S. Ergun, Fluid flow through packed columns, *Chem. Eng. Prog.*, **48**, 89–94 (1952).
4. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, *Heat Transfer* [in Russian], Énergoizdat, Moscow (1981).
5. V. I. Kovenskii and Yu. S. Teplitskii, On the thermal conductivity of a blown granular bed, *Inzh.-Fiz. Zh.*, **81**, No. 5, 956–962 (2008).
6. N. I. Gel'perin, V. G. Ainshtein, and V. B. Kvasha, *Principles of Fluidization Technique* [in Russian], Khimiya, Moscow (1967).
7. Yu. S. Teplitskii and V. I. Kovenskii, Heat transfer in granular beds with radiative heat supply, *Inzh.-Fiz. Zh.* (in press).